Measurement Excellence™

BERTEC CORPORATION

Force Plate Load Calculations & Coordinate System Changes

Calculating Load Values

Each force plate is calibrated individually and the calibration matrix is stored digitally in the force plate. Therefore, the analog output from the amplifier provides full-scale calibrated output (±5 V) per rated load range of the attached force plate. The voltage output of each channel is a scaled form of the load in the units of N and N·m for the forces and moments respectively. The scale factor for each channel for a gain of unity is given in the product data sheet supplied with the transducer. The force and moment values are calculated by multiplying the signal values with corresponding scale factors, as given in Eqn. 1:

$$\begin{aligned} F_x &= C_1 \cdot S_1 \\ F_y &= C_2 \cdot S_2 \\ F_z &= C_3 \cdot S_3 \\ M_x &= C_4 \cdot S_4 \\ M_y &= C_5 \cdot S_5 \\ M_z &= C_6 \cdot S_6 \end{aligned} \qquad (1)$$

where, F's and M's are the force and moment components in the force transducer coordinate system (Figure 1), and S's are the output signals corresponding to the channels indicated by their subscripts, in volts, divided by the respective channel gain. The origin of the coordinate system is centered on the top surface of the force plate (see Figure 1). The standard coordinate system is such that the positive y-direction is opposite to the connector end; x-axis is to the left when looking in the y-axis direction; and the z-axis is defined downwards by the right hand rule.

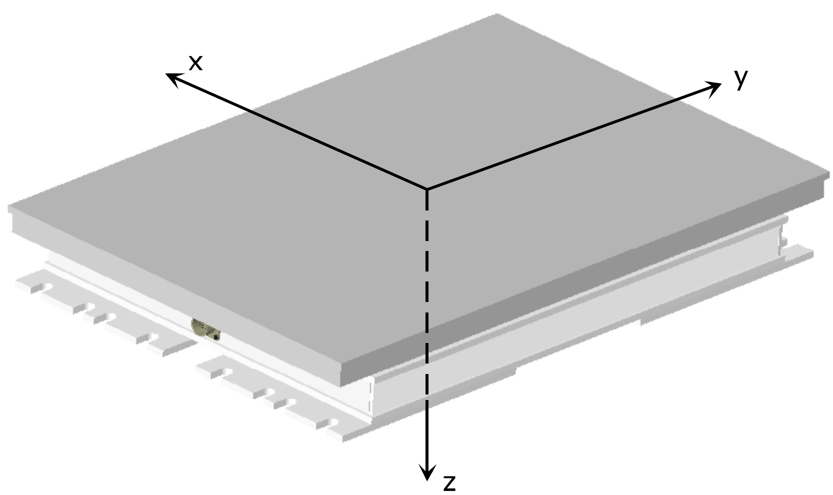

Figure 1– Standard force plate coordinate system: the origin is on the top surface, and at the center of the plate. Positive y-direction is opposite to the connector end; x-axis is to the left when looking in the y-axis; and the z-axis is defined downwards by the right hand rule.

Calculation of Point of Application for the Force and Couple

A load system acting on a force plate can be completely described by the six load components (i.e. the three force and three moment components) calculated from Eqn. 1. Alternatively, the same information can be given as the three force components, the point of application of the force vector (x_p, y_p in Figure 2), and a couple (sometimes also referred as "torque " or "free moment") acting on the force plate. Referring to Figure 2, the point of application of the force, and the couple are calculated from the force and moment components as:

$$x_p = \frac{-h \cdot F_x - M_y}{F_z}$$
$$y_p = \frac{-h \cdot F_y + M_x}{F_z} \qquad (2)$$
$$T_z = M_z - x_p \cdot F_y + y_p \cdot F_x$$

where, x_p and y_p are the coordinates of the point of application for the force (i.e. center of pressure); h is the thickness, above the top surface, of any material covering the force plate (Figure 2); and T_z is the couple acting on the force plate. Note that the thickness h, shown in Figure 2, is to be entered as a positive number in Eqn. 2.

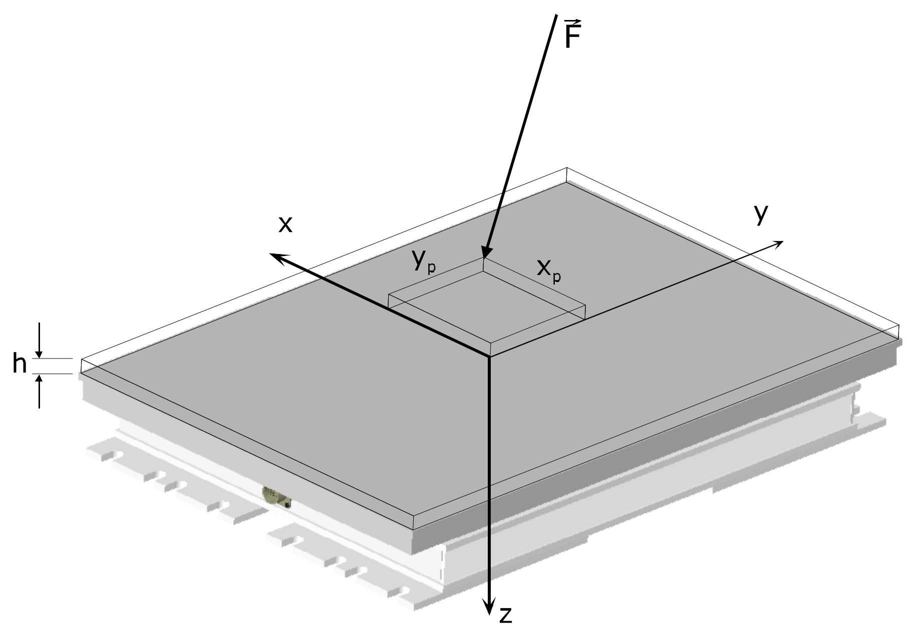

Figure 2 – A force F, and the point of application of the force. The force plate is covered with a layer of floor covering, which has a thickness ***h***. The thickness ***h*** is entered as positive number in Eqn. 2.

Example 1: Load Computation

Consider a case where the external amplifier gain is set to 10 (note that the gain value is always the same for all of the six channels). If, at an instant in time, the amplifier voltage outputs for the six channels are:

CHANNEL	OUTPUT, V
1	-1.450
2	2.235
3	4.765
4	3.095
5	-0.575
6	-1.016

Then, by dividing each output by the corresponding gain, the output signal values to be used in Eqn. 1 are obtained:

$$S_1 = -1.450/10 = -0.145 \text{ V}$$

$$S_2 = 2.235/10 = 0.2235 \text{ V}$$

$$S_3 = 4.765/10 = 0.4765 \text{ V}$$

$$S_4 = 3.095/10 = 0.3095 \text{ V}$$

$$S_5 = -0.575/10 = -0.0575 \text{ V}$$

$$S_6 = -1.016/10 = -0.1016 \text{ V}$$

Let us use hypothetical scale factors, in N/V and N·m/V[1]:

$$C_1 = 1000 \text{ N/V}$$

$$C_2 = 1000 \text{ N/V}$$

$$C_3 = 1500 \text{ N/V}$$

$$C_4 = 300 \text{ N·m/V}$$

$$C_5 = 300 \text{ N·m/V}$$

$$C_6 = 250 \text{ N·m/V}$$

Then from Eqn. 1:

[1] Note that if the results are needed in English Units, an alternative to converting them at the end of calculations is to convert the scale factors to English Units by converting the first three factors from N/V to lb/V, and the last three factors from N·m/V to ft·lb/V. This can be done by multiplying the first three scale factors by 0.2248 lb/N, and last three scale factors by 0.7377 (ft·lb)/(N·m).

$F_x = 1000 \cdot (-0.145) = -145.0$ N

$F_y = 1000 \cdot (0.2235) = 223.5$ N

$F_z = 1500 \cdot (0.4765) = 714.8$ N

$M_x = 300 \cdot (0.3095) = 92.9$ N·m

$M_y = 300 \cdot (-0.0575) = -17.3$ N·m

$M_z = 250 \cdot (-0.1016) = 25.4$ N·m

To calculate the point of application of the force, Eqn. 2 is used. Assuming there is a 5 mm covering on the top surface of the transducer, then *h*=0.005 m. Therefore:

$$x_p = \frac{(-0.005)\cdot(-145.0) + 17.3}{714.8} = 0.025\,\text{m}$$

$$y_p = \frac{(-0.005)\cdot(223.5) + 92.9}{714.8} = 0.128\,\text{m}$$

Change of Coordinate System

In numerous applications measurement protocols require that the forces and moments be measured with respect to a coordinate system other than the force plate's local coordinate system shown in Figure 1. This secondary coordinate system might be that of a motion analysis system or it might belong to another force plate. In such a case the components of force and moment vectors should be expressed in this secondary system. For this purpose, the exact location and orientation of the secondary coordinate system with respect to the force plate local system should be known. For the case shown in Figure, coordinate system 1 is the force plate's local coordinate system, and a secondary system 2 is located so that its axes are rotated and displaced in 3-dimensional space. The rotational displacement is such that the angle between axes are given in terms of angles θ_{11}, θ_{12}, … θ_{33}, where θ_{ij} (i=1, 2, 3; j=1, 2, 3) is the angle between the unit vectors $\vec{u}_i^1$ and $\vec{u}_j^2$ of the two coordinate systems shown in Figure. The displacement of the origin of 1 with respect to 2 is given as the vector $\vec{r} = \{r_1 \quad r_2 \quad r_3\}$, where r_1, r_2 and r_3 are measured in the second coordinate system. The measured forces and moments can be transformed to coordinate system 2 using the following relations:

$$\begin{Bmatrix} F_x^2 \\ F_y^2 \\ F_z^2 \end{Bmatrix} = [T] \cdot \begin{Bmatrix} F_x^1 \\ F_y^1 \\ F_z^1 \end{Bmatrix} \tag{3}$$

$$\begin{Bmatrix} M_x^2 \\ M_y^2 \\ M_z^2 \end{Bmatrix} = [T] \cdot \begin{Bmatrix} M_x^1 \\ M_y^1 \\ M_z^1 \end{Bmatrix} + \vec{r} \times \begin{Bmatrix} F_x^2 \\ F_y^2 \\ F_z^2 \end{Bmatrix} \tag{4}$$

where, superscript "1" denotes measured quantities, superscript "2" indicates the same quantities expressed in coordinate system 2, and [*T*] is a transformation matrix computed using the θ_{ij} values described above. The elements of the 3x3 transformation matrix are the direction cosines of the coordinate axes arranged as:

$$[T] = \begin{bmatrix} \cos\theta_{11} & \cos\theta_{12} & \cos\theta_{13} \\ \cos\theta_{21} & \cos\theta_{22} & \cos\theta_{23} \\ \cos\theta_{31} & \cos\theta_{32} & \cos\theta_{33} \end{bmatrix}$$

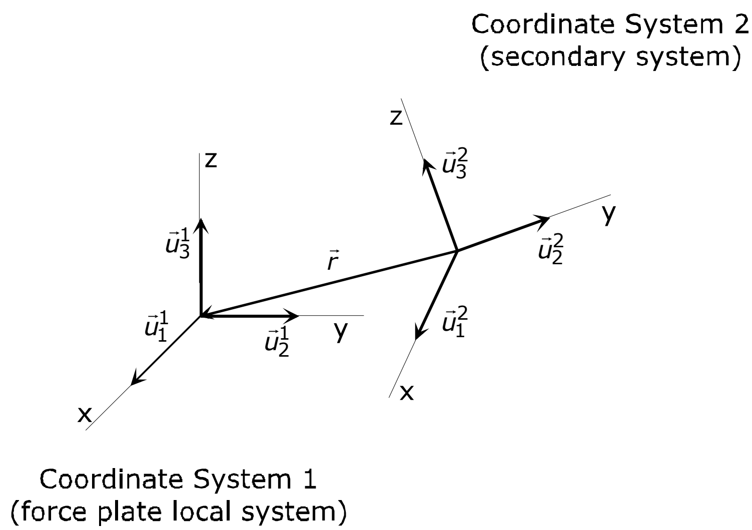

Figure 3 - Coordinate system 1 is a force plate's local coordinate system in which the loads are measured. The secondary coordinate system is displaced and rotated with respect to the first in 3-dimensional space.

Example 2: Change of Reference System

Let's assume that in a gait analysis laboratory the ground reaction forces and moments are measured in the force plate local coordinate system with the axes x_f, y_f, z_f shown in Figure 3. The motion analysis system, however, requires these loads to be computed in a laboratory fixed coordinate system located at the corner of the force plate with the axes x_L, y_L, z_L. oriented as given in Figure 3. The x and z-axes of both coordinate systems are pointing in opposite directions rotated by 180º, and the y-axes are parallel to each other. The origins are displaced by 20 cm in x-direction, and 30 cm in y-direction. For such an arrangement the vector $\vec{r}$ will be {0.2 0.3 0} m. Since the corresponding coordinate axes are parallel to each other we have the following values for the angles θ:

$$\theta_{11} = 180º,\ \theta_{22} = 0º,\ \theta_{33} = 180º$$

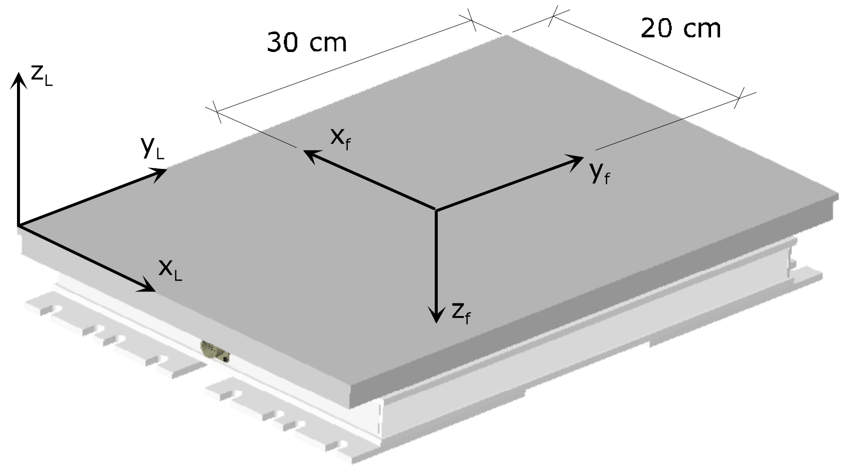

Figure 3 – The ground reaction load is measured in the force plate's local coordinate system denoted by the subscript "f". Then the components of the force and moment vectors are transferred to the laboratory coordinate system indicated by the subscript "L".

The rest of the angles are either 90º or -90º. Using these values the transformation matrix is calculated as:

$$[T] = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Using the hypothetical measured values calculated in Example 1 above in Equations 3 and 4, we get

$$\begin{Bmatrix} F_x^2 \\ F_y^2 \\ F_z^2 \end{Bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \cdot \begin{Bmatrix} -145.0 \\ 223.5 \\ 714.8 \end{Bmatrix} = \begin{Bmatrix} 145.0 \\ 223.5 \\ -714.8 \end{Bmatrix} \mathrm{N}$$

$$\begin{Bmatrix} M_x^2 \\ M_y^2 \\ M_z^2 \end{Bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \cdot \begin{Bmatrix} 92.9 \\ -17.3 \\ 25.4 \end{Bmatrix} + \begin{Bmatrix} 0.2 \\ 0.3 \\ 0 \end{Bmatrix} \times \begin{Bmatrix} 145.0 \\ 223.5 \\ -714.8 \end{Bmatrix}$$

$$\begin{Bmatrix} M_x^2 \\ M_y^2 \\ M_z^2 \end{Bmatrix} = \begin{Bmatrix} -307.3 \\ 125.7 \\ -24.2 \end{Bmatrix} \mathrm{N} \cdot \mathrm{m}$$